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(Quasi)Localized Gauge Field on a Brane: Dissipating Cosmic Radiation to Extra Dimensions?

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Abstract

We propose a mechanism ensuring (quasi)localization of massless gauge fields on a brane. The mechanism does not rely on BPS properties of the brane and can be realized in any theory where charged particles are confined to the worldvolume. The localized matter fluctuations induce a gauge kinetic term on the brane. At short distances the resulting propagator for the gauge field is *identical* to the four-dimensional propagator. The gauge theory on the brane is effectively four-dimensional at short distances; it becomes higher-dimensional on very large (cosmic) scales. The brane-bulk system exhibits the phenomenon of "infrared transparency". As a result, only very low frequency modes can escape into extra dimensions. In this framework the large wavelength cosmic radiation can dissipate in extra space at a rate that may be observable, in principle. We briefly discuss some astrophysical consequences of this scenario.

The same mechanism of localization of gauge fields may work in Kaplan's framework for domain wall chiral fermions on lattices.

1 Introduction

The idea that the Standard Model particles can be localized on a brane may dramatically change the low-energy implications of extra dimensions and quantum gravity. In conjunction with simple compactifications, this idea allows for extra dimensions to be macroscopically large ($\sim 10^0$ mm or so), with the fundamental quantum gravity scale being as low as a few TeV, thus providing an alternative view on the hierarchy problem [1]. Noncompact, but finite-volume extra dimensions are also possible if one introduces a negative bulk cosmological constant [2].

Putting aside the hierarchy problem, the brane world idea may have other implications, e.g., it can give rise to a natural mechanism for supersymmetry breaking. If the brane at hand is a non-BPS state then it breaks all supercharges of the bulk space rendering the four-dimensional world free of the Fermi-Bose degeneracy [3]. Moreover, one may naturally explain in this way the smallness of the SUSY-breaking parameters [4].

In the brane world model-building the question of paramount importance is the localization of the Standard Model particles on the brane. A mechanism of the fermion localization was suggested long time ago in [5], based on an index theorem of Ref. [6]. The localization mechanisms for scalar and fermion fields in warped backgrounds are known as well [7] (see also [8]).

The vital issue is how vector fields can be localized on the brane. This question will be the main concern of the present paper. The gauge fields can be localized on D-branes very naturally in the framework of string theory [9]. D-branes are BPS objects that preserve half of the original supersymmetry; their low-energy world-volume theory is described by massless fluctuations of open strings ending on them. In fact, the localized vector fields on a D-brane world-volume are just spin-1 massless fluctuations of the corresponding open strings. Unbroken supersymmetry then guarantees that these spin-1 fluctuations stay massless in order to correctly fill up the supermultiplets together with the massless world-volume scalars and fermions.

However, in the brane world context it is highly desirable that the brane is not a BPS state since supersymmetry must be broken in a world-volume theory. Whether or not the same mechanism of localization of the vector fields holds for non-BPS branes, especially those which cannot be presented as bound states of BPS states, is not clear. In this respect it is crucial to develop additional field-theoretic mechanisms for the massless spin-1 localization on a brane, which would be independent of BPS properties.

One such mechanism was proposed in Ref. [10], where the gauge theory is assumed to be confining in the bulk but not on the brane. As a result the electric charges on the brane are not screened, but rather are in a Coulomb phase since the electric flux cannot spread in the bulk. Thus, there is a massless gauge field trapped on the brane world-volume theory. In this case neither photon nor electric charges can escape to extra dimensions. Another possibilities were recently proposed in Refs. [11] and [12] where the gauge fields were localized on the brane in extra-dimensional

space which was warped (and in addition compactified in [12]).

In this work we will present a mechanism of (quasi)localization of gauge fields on a brane which is rather model-independent. This mechanism can be used in theories with compact, warped and, most importantly, infinite-volume extra dimensions. This is a generalization of the mechanism of Ref. [13] for the localization of a massless spin-2 field on the brane due to a "brane-induced" world-volume Ricci term. The main idea of Ref. [13] is as follows: Consider a bulk field coupled minimally to some of the brane-localized degrees of freedom. Then, a four-dimensional kinetic term for a given bulk field is generated on the brane world-volume due to loop corrections associated with localized matter fields [13]. This brane-induced kinetic term ensures that the propagation of the bulk field on the brane is *four-dimensional* at the distances smaller than a certain crossover scale r_c. In this way brane "induces" an effectively four-dimensional theory in its world-volume. In Ref. [13] this idea was used for the (quasi)localization of the massless spin-2 field on a brane. In this case the four-dimensional laws of gravity can be obtained on a *delta-function* type brane even if extra dimensions have infinite volume [13, 14]. This mechanism is quite universal since it requires only the presence of localized spinor and/or scalar fields on the brane. Moreover, the same mechanism is valid even if there are no localized matter fields on the brane but the brane has a finite thickness. In the latter case the four-dimensional Ricci scalar is induced in the low-energy world-volume theory after the effects of the finite brane thickness are integrated out [14]. In this framework there is no need to compactify or to warp the extra space, it can have an infinite volume [13, 14].

In the present work we will generalize the above mechanism for spin-1 fields. For this mechanism to work it is sufficient to have bulk gauge fields and a brane on which charged matter fields are localized. Then the quantum fluctuations of localized matter automatically take care of (quasi)localization of the bulk gauge fields. To be specific, in the present work we deal with five-dimensional models. Higher dimensions can be treated in a similar manner. In Sect. 2, we consider an idealized case of a five-dimensional theory with a delta-function type brane. This simplification is temporary as in Sect. 3 we consider a field-theoretic setting with a finite-thickness brane [1]. The field-theoretic consideration is instructive since it allows us to deduce an exact expression for a tree-level four-dimensional potential mediated by the quasilocalized gauge fields. Moreover, we note that quasilocalization on the zero-thickness brane can be applied to objects which can be treated as singular sources in the weak coupling approximation of string theory. Such are D-branes, orientifold planes and/or extended objects which are stuck to the orbifold fixed points[2] (all at weak coupling).

[1]In those cases when there is a menace of confusion, we will refer to the field-theoretic setting with the finite-thickness brane as to a domain wall, or wall for short, reserving the term "brane" for the object which has zero transverse width in a classical approximation.

[2]This statement needs some qualifications since in Type IIA,B string theories in ten dimensions there are no bulk gauge fields in the first place. However, bulk gauge fields can emerge once some

In Sect. 3 we consider a five-dimensional field theory with a domain wall soliton in which we *derive* the results of Sect. 2 in the case of a finite-thickness wall. In fact, we will show that the domain wall with the localized fermion zero-modes produces a four-dimensional kinetic term for the gauge fields and, thus, leads to quasilocalization of spin-1 particles.

Finally, in Sect. 4 we analyze some phenomenological consequences of quasilocalization. Since now the gauge fields can escape in the bulk, local charges in the world-volume are not necessarily conserved [12]. A very intriguing feature of the framework is that the large wavelength cosmic radiation can dissipate into the bulk over the cosmic distances, by the rate that is, in principle, detectable. This offers a number of exciting astrophysical implications for the above scenario, which we shall briefly discuss.

2 Induced Gauge Fields on a Brane

In this section we consider a simple case – a brane of the *delta-function* type embedded in five-dimensional space. In other words, we will disregard effects due to non-vanishing thickness of the wall.

We will not be interested in effects due to gravity. We rather assume that the brane at hand is made of some heavy fields which are already decoupled from the low-energy dynamics (an explicit field-theoretic realization will be give in the next section).

Suppose that there are some gauge fields $\mathcal{A}_C$ (here $C = 0, 1, 2, 3, 5$) living in the bulk for which the Lagrangian takes the form

$$-\frac{1}{4g^2}\,\mathcal{F}^2_{AB}\ + \text{other fields}, \tag{1}$$

where g is a coupling constant with the dimensionality

$$[g^2]\ =\ [\text{mass}]^{-1}\ . \tag{2}$$

For simplicity we can assume that $\mathcal{A}_C$ is an Abelian field; similar considerations can be carried out in the non-Abelian case. The three-brane on which matter is localized can be put at the point $y = 0$.

The Dirac-Nambu-Goto action for a brane takes the form:

$$S_{\text{3-brane}}\ =\ -T\ \int d^4x\ \sqrt{|\text{det}\bar{g}|}\,, \tag{3}$$

where T stands for the brane tension, and

$$\bar{g}_{\mu\nu} = \partial_\mu X^A \partial_\nu X^B G_{AB}$$

of the extra six dimensions are compactified. For instance, in compactification to five dimensions on $K3 \times S^1$ there are points of enhanced gauge symmetry on the moduli space where the gauge fields emerge. Thus, we can have a brane with localized matter as well as the bulk populated by the gauge fields.

denotes the induced metric on the brane (with G_{AB} being the metric of five-dimensional space-time). X^A $(A = 1, 2, ..., 5)$ are the coordinates in 5D space. If for simplicity we neglect brane's fluctuations (these are just scalar particles which can easily be included in the consideration if so desired) then $\partial_\mu X_5 = 0$, and $X_\mu = x_\mu$ so that the induced metric can be written as follows:

$$\bar{g}_{\mu\nu}\,(x) \;=\; G_{\mu\nu}\,(x,y)\big|_{y=0}\;.$$

In general, there could be localized matter fields on the brane world-volume. One can take them into account by writing the following action:

$$\tilde{S}^{\rm matter}_{\text{3-brane}} = S_{\text{3-brane}} + \int d^4x\,\sqrt{|{\rm det}\bar{g}|}\;\tilde{\mathcal{L}}(\psi)\,, \tag{4}$$

where ψ denotes collectively all localized fields for which the four-dimensional Lagrangian density is $\tilde{\mathcal{L}}$. The world-volume field theory may be regarded as an effective field theory with some cutoff Λ. The current of the localized matter fields on a brane can be written as follows:

$$J_A(x,y) \;=\; J_\mu(x)\;\delta(y)\;\delta^\mu_A\,. \tag{5}$$

The five-dimensional current conservation associated with the five-dimensional gauge invariance, implies the four-dimensional current conservation on the brane, since the fifth component of the current vanishes. This current couples to the bulk gauge field. The interaction Lagrangian is

$$\mathcal{L}_{\rm int} \;=\; \int d^4x\,dy\,J_C(x,y)\,\mathcal{A}^C(x,y) = \int d^4x\;J_\mu(x)\;\mathcal{A}^\mu(x,0)\,. \tag{6}$$

Thus, the current of localized matter interacts with the gauge field

$$A_\mu(x) \equiv \mathcal{A}(x, y=0)\,. \tag{7}$$

Due to the interaction (6) an induced kinetic term for the field $A_\mu(x)$ emerges on the brane world-volume. This term is generated by a one-loop diagram with two external legs of $A_\mu(x)$ and localized matter running in the loop. The four-dimensional current conservation implies transversality of this loop; as a result, the following term should be included in the total low-energy action on the brane

$$-\frac{1}{4e^2}\,F^2_{\mu\nu} + \text{higher derivative terms}\,, \tag{8}$$

where

$$e^{-2} = \frac{2N_f}{3\pi}\ln\frac{\Lambda}{\mu}\,,$$

Λ and μ are the ultraviolet and infrared cut-offs, respectively. The sign of the induced term is negative, as it should be, since it is generated by localized scalar

and/or spinor fields (confined to the brane world-volume) running in the loops. Were the loops generated by localized vector fields, the sign would be wrong (positive).

The total (five-dimensional) Lagrangian for the gauge fields takes the form

$$-\frac{1}{4g^2}\,\mathcal{F}_{AB}^2-\frac{1}{4e^2}\,F_{\mu\nu}^2\,\delta(y)+\text{other interactions}\,. \tag{9}$$

Below we will study the impact of the induced kinetic term in (9) at the tree-level.

Our first task is to calculate the Coulomb potential induced by a probe charged particle placed on the brane. To this end, we add a source term $\mathcal{A}^B J_B$ to the Lagrangian (9). The equation of motion in the theory with the source is

$$\frac{1}{g^2}\,\partial_C\partial^C\mathcal{A}_B+\delta(y)\frac{1}{e^2}\left(\partial_\mu\partial^\mu\mathcal{A}_\nu+\partial_\nu(\partial_y\mathcal{A}_y)\right)\,\delta_{\nu B}\;=\;J_B(x,y)\,, \tag{10}$$

where we have chosen the Lorentz gauge in the bulk

$$\partial^C\mathcal{A}_C=0\,. \tag{11}$$

Next, we assume the source to be localized in the y direction (see Eq. (5)) to transform Eq. (10) into

$$\begin{aligned}&\partial_C\partial^C\,\mathcal{A}_\mu\;+\;\frac{g^2}{e^2}\delta(y)\,\Big[\partial_\beta\partial^\beta\mathcal{A}_\mu\;+\;\partial_\mu(\partial_y\mathcal{A}_y)\Big]=g^2\,J_\mu(x)\delta(y)\,,\\&\partial_C\partial^C\,\mathcal{A}_y=0\,.\end{aligned} \tag{12}$$

To derive the propagator for the gauge field $A_\mu(x)$ it is convenient to Fourier-transform these equations with respect to the four world-volume coordinates. The corresponding momenta will be denoted by p; we will work in Euclidean space,

$$p^2=p_4^2+p_1^2+p_2^2+p_3^2\,.$$

Equations (12) can be written as follows:

$$\begin{aligned}&(p^2-\partial_y^2)\tilde{\mathcal{A}}_\mu(p,y)+\frac{g^2}{e^2}\,\delta(y)\,\Big(p^2\tilde{\mathcal{A}}_\mu(p,y)+\,ip_\mu\partial_y\tilde{\mathcal{A}}_y(p,y)\Big)\\&=g^2\,\tilde{J}_\mu(p)\,\delta(y)\,,\end{aligned} \tag{13}$$

$$(p^2-\partial_y^2)\,\tilde{\mathcal{A}}_y(p,y)=0\,, \tag{14}$$

where the tilde denotes the Fourier transform. Equation (13) can be readily solved. Multiplying both sides by $\tilde{J}_\mu(p)$ and using the four-dimensional transversality (i.e. $p^\mu\tilde{J}_\mu(p)=0$) we obtain

$$\tilde{\mathcal{A}}_\mu(p,y)\tilde{J}^\mu(p)=e^2\left[\tilde{J}_\mu(p)\right]^2\frac{1}{p^2+(2pe^2/g^2)}\;\exp\left(-p|y|\right)\,,\qquad p\equiv\sqrt{p^2}\,. \tag{15}$$

It describes the gauge field with somewhat unusual propagator, more of which will be said later.

Moreover, Eq. (10) combined with the four-dimensional transversality of the current implies that $\partial^\mu \mathcal{A}_\mu(x,y) = 0$, which entails, in turn, from the gauge fixing condition (11) that

$$\partial_y \mathcal{A}_y(x,y) = 0\,. \tag{16}$$

Combining this with (14) we get the following equation for the fifth component of the bulk gauge field:

$$\partial_\mu^2 \mathcal{A}_y(x,y) = 0\,. \tag{17}$$

The latter equation describes a four-dimensional massless scalar field which is decoupled from the matter fields localized on the brane [3]. Therefore, from the point of view of a four-dimensional observer we can forget about $\mathcal{A}_y$ for the time being, and focus on A_μ.

The propagator of the gauge field on the brane world-volume at $y = 0$ takes the form

$$D_{\mu\nu}(p) = \frac{\eta_{\mu\nu}}{p^2 + (2e^2 p/g^2)}\left[1 + \mathcal{O}(p)\right]\,. \tag{18}$$

Physics of this type of propagators was elucidated in Ref. [13]. There are two distinct regimes (a critical value of momentum is $p_* = 2e^2/g^2$). For small momenta, $p \ll p_*$ (i.e. at large distances) the propagator resembles that of the five-dimensional theory. However, at large momenta, $p \gg p_*$ (i.e. at small distances) it behaves as the four-dimensional propagator. The crossover scale is determined by the bulk coupling constant g^2.

It is instructive to study these properties in more detail in the coordinate space. For static probe charges only the time component of the current survives; it is not difficult to calculate the tree-level scalar potential due to the exchange of the vector field (18). Using the results of Ref. [13] we get

$$V(r) \;\propto\; \frac{1}{r}\;\left\{ \sin\left(\frac{r}{r_c}\right)\;\mathrm{Ci}\left(\frac{r}{r_c}\right) \;+\; \frac{1}{2}\cos\left(\frac{r}{r_c}\right)\left[\pi \;-\; 2\;\mathrm{Si}\left(\frac{r}{r_c}\right)\right]\right\}\;, \tag{19}$$

where

$$\mathrm{Ci}(z) \equiv \gamma + \ln(z) + \int_0^z (\cos(t) - 1)dt/t\,, \qquad \mathrm{Si}(z) \equiv \int_0^z \sin(t)dt/t\,,$$

$\gamma \simeq 0.577$ is the Euler-Mascheroni constant, and the distance scale r_c is defined as follows:

$$r_c \equiv \frac{g^2}{2e^2}\,. \tag{20}$$

[3]Effects due to interactions of this scalar with the matter will be considered in the next section.

Let us examine more carefully the short- and long-distance behavior of this expression.

At short distances when $r << r_c$ we find:

$$V(r) \;\propto\; \frac{1}{r}\;\left\{\frac{\pi}{2}+\left[-1+\gamma+\ln\left(\frac{r}{r_c}\right)\right]\left(\frac{r}{r_c}\right)\;+\;\mathcal{O}(r^2)\right\}\;. \tag{21}$$

As was expected, at short distances the potential has the correct four-dimensional $1/r$ scaling. At intermediate distances it is modified by a logarithmic *repulsion* term.

Now let us turn to the large-distance behavior. Using (19) we obtain for $r \gg r_c$

$$V(r) \;\propto\; \frac{1}{r}\;\left\{\frac{r_c}{r}\;+\;\mathcal{O}\left(\frac{1}{r^2}\right)\right\}\;. \tag{22}$$

Thus, the large-distance potential scales as $1/r^2$, in full accordance with the five-dimensional theory laws.

The physical interpretation of this crossover phenomenon is as follows. A gauge field emitted by the source localized on the brane propagates along the brane but gradually dissipates in the bulk. The lower the frequency of the signal the faster it leaks in the extra space (this is the phenomenon of "infrared transparency"). This is similar to what happens with the metastable gravitons in the models of Refs. [15, 16, 17]. Since in the model at hand the bulk gauge coupling is a free parameter, its value should be fixed by experimental bounds on the photon propagation in four dimensions (see below).

Returning to the momentum representation we observe that in Minkowski the propagator (18) develops a discontinuity across the cut. Denoting the Minkowski (four-)momentum squared by p_M^2 one readily gets both the imaginary and real parts of the photon propagator on the cut (i.e. at $p_M^2 > 0$),

$$\mathrm{Re}\, D(p) = \frac{1}{p_M^2+p_*^2}\,, \qquad \mathrm{Im}\, D(p) = \frac{p_*}{p_M}\;\frac{1}{p_M^2+p_*^2}\,, \tag{23}$$

where we suppressed the tensorial structure of $D_{\mu\nu}$. In Sect. 4 we will discuss effects due to the deviation of (23) from the Feynman propagator at small momenta. Note that the imaginary part is suppressed with regards to the real part by one power of p_*/p_M.

3 Spontaneous Quasilocalization on Finite-Thickness Domain Wall

In this section we will consider a field-theoretic model resulting in a finite-thickness domain wall and analyze how quasilocalization of the gauge field works in this case. The main emphasis is put on effects which are invisible in the zero-width limit.

Our starting point is the five-dimensional model with scalars, spin-1/2 fermions and the vector fields in the bulk. The Lagrangian has the form

$$\mathcal{L} = -\frac{1}{4g^2}\,\mathcal{F}_{AB}^2 + i\bar{\Psi}\gamma^C(\partial_C + i\mathcal{A}_C)\Psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{2}(\phi^2 - v^2)^2 + \phi\,\bar{\Psi}\Psi\,, \tag{24}$$

where ϕ is a real scalar field, and Ψ is a five-dimensional spinor. From the point of view of a four-dimensional observer Ψ is a set of two Weyl spinors, one dotted and one undotted, $\Psi = \{\xi, \bar{\eta}\}$. Moreover, $(\gamma^5)_{5\mathrm{D}} = -i\,(\gamma^5)_{4\mathrm{D}}$.

The equation of motion for the scalar field ϕ has a well-known kink solution,

$$\phi_0(y) = v\tanh\left(\sqrt{\lambda}vy\right)\,. \tag{25}$$

The corresponding domain wall provides a toy model for our four-dimensional world.

To begin with, let us put the vector field $\mathcal{A}_C = 0$. There are fermion zero modes localized on the domain wall, i.e. normalizable solutions of the equation $i\,(\gamma^5)_{5\mathrm{D}}\,\partial_5\Psi + \phi_0\Psi = 0$. For the wall configuration (25) a normalizable solution exists for ξ, while for antikink a normalizable solution exists for η, i.e. the zero modes are chiral [6].

From the standpoint of a four-dimensional observer the only fermionic mode which is light is

$$\Psi(x,y) = \{\xi(x)\,f(y),\ 0\}\,, \qquad f(y) \equiv \mathcal{N}\exp\left(-\int_0^y \phi_0(z)dz\right)\,, \tag{26}$$

where $\xi_\alpha(x)$ is the four-dimensional fermion field ($\alpha = 1,2$); it is left-handed. Moreover, $\mathcal{N}$ is a normalization factor introduced in such a way that $\int dy f^2(y) = 1$.

All other modes acquire masses of order $\sqrt{\lambda}v$. In the low-energy approximation these heavy modes should be integrated out. The result of this procedure is well-known, it gives rise to higher-dimensional operators which are suppressed by powers of v. The only remnant which is important is a Chern-Simons term which could have been anticipated *a priori*. Indeed, while five-dimensional electrodynamics of the field Ψ is well defined, this is not the case for four-dimensional electrodynamics of a chiral field ξ; it is anomalous. The Chern-Simons term eliminates this anomaly.

The five-dimensional Chern-Simons term generated in the bulk [18] has the form

$$k\,\epsilon^{ABCDE}\mathcal{A}_A\mathcal{F}_{BC}\mathcal{F}_{DE}\,, \tag{27}$$

where k is some dimensionless constant. This term provides anomaly inflow from the bulk in the theory living on the wall so that the axial anomaly due to the coupling of the localized chiral zero-modes to the vector field is canceled by the surface term emerging from the variation of the Chern-Simons term in (27) [18][4].

[4] In general, one could introduce N_f flavors with different charges so that the anomaly in the worldvolume theory is canceled.

Having said this, let us return to the localized massless modes on the wall, and switch on the gauge field. From the standpoint of a four-dimensional observer $\mathcal{A}_\mu$ ($\mathcal{A}_y$) is a vector (scalar) field. The latter is not coupled to the zero mode. This follows from the fact that the only 4D scalar bilinear one can write is $\xi_\alpha\xi^\alpha$ and its Hermitian conjugate. This combination is charged, however, with respect to the conserved U(1) present in (24) and, thus, cannot appear. Thus, the chirality of the zero modes prevent nonderivative couplings of $\mathcal{A}_y$ to localized matter.

The coupling of the zero mode to the bulk vector field is as follows:

$$\bar{\xi}_{\dot{\alpha}}(x)\xi_\alpha(x)(\sigma^\mu)_{\dot{\alpha}\alpha}\int dy\,\mathcal{A}_\mu(x,y)\ f^2(y) \equiv \bar{\xi}_{\dot{\alpha}}(x)\xi_\alpha(x)(\sigma^\mu)_{\dot{\alpha}\alpha}\,a_\mu(x)\,, \tag{28}$$

where we defined a new four-dimensional vector field $a_\mu(x)$ as

$$a_\mu(x) = \int dy\,\mathcal{A}_\mu(x,y)\ f^2(y).$$

The four-dimensional chiral fermions are coupled to $a_\mu(x)$ in the standard manner, in the way the charged fermions should be coupled to the U(1) gauge field. This coupling necessarily induces a gauge kinetic term for the vector-potential $a_\mu(x)$, via the fermion loop with two external $a_\mu(x)$ legs, much in the same way as in Eq. (8), with the substitution of A_μ of Eq. (7) by a_μ. The coefficient in front of this term has the right sign.

The total low-energy action for the fermions and gauge fields is the sum of the bulk kinetic term (1), the Chern-Simons term (27), the induced term

$$-\frac{1}{4e^2}\left(\partial_\mu a_\nu - \partial_\nu a_\mu\right)^2\,, \tag{29}$$

and the interaction term (28). Besides, there is a massless Goldstone boson and the massless $\mathcal{A}_y$ field in the world-volume. The former interacts with fermions with derivatives and does not contribute to the Coulomb potential between the localized sources. The latter field has no relevant interactions with localized fields in the given model[5].

Let us now study the Coulomb force between the localized sources on the brane. This problem is quite similar to the one studied in the previous section. There are two modes of propagations of the vector field between any two points on the brane. One is a bulk kinetic term for the gauge fields and the other one is the induced kinetic term. As before, choosing the Lorentz gauge in the bulk we write equations of motion analogous to (12),

$$\begin{aligned}
&\partial_C\partial^C\mathcal{A}_\mu(x,y) + \frac{g^2}{e^2}f^2(y)\left[\partial_\nu\partial^\nu\mathcal{A}_\mu\ +\ \partial_\mu(\partial_y\mathcal{A}_y)\right]\ =\ g^2 J_\mu(x)\delta(y)\,,\\
&\left(\partial_\mu\partial^\mu\ +\ \partial_y^2\right)\mathcal{A}_y\ =\ 0.
\end{aligned} \tag{30}$$

[5]There are interaction vertices of the $\mathcal{A}_y$ scalar with one light and one heavy state. Thus, $\mathcal{A}_y$ can be produced by a light state in high energy processes with the energy of order $\sim v$.

Making a (partial) Fourier transformation to the momentum space, as defined in the previous section, we arrive at

$$(p^2-\partial_y^2)\,\tilde{\mathcal{A}}_\mu(p,y)+\frac{g^2}{e^2}f^2(y)\left[p^2\ \tilde{\mathcal{A}}_\mu(p,y)+ip_\mu\partial_y\tilde{\mathcal{A}}_y\right]=g^2\,\tilde{J}_\mu(p)\delta(y)\,. \tag{31}$$

The function $f(y)$ is peaked and quickly drops down to zero away from the origin. To solve this equation we approximate it by a step-function *ansatz*,

$$f(y)=\alpha \quad \text{for} \quad |y|<y_0, \qquad f(y)=0 \quad \text{for} \quad |y|>y_0, \tag{32}$$

where y_0 is some positive constant. The delta-function approximation studied in the previous section is obtained in the limit

$$y_0\to 0, \quad \alpha\to\infty, \qquad \text{with} \ \ y_0\alpha \ \ \text{const.} \tag{33}$$

Equation (31) can be solved piecewise in three different intervals. The solutions are

$$\tilde{\mathcal{A}}_\mu(p,y)=g^2\ \tilde{J}_\mu(p)\left\{\frac{1}{2\sqrt{\beta}p}e^{-p\sqrt{\beta}|y|}\ +\ b\ e^{-p\sqrt{\beta}y}\ +\ d\ e^{p\sqrt{\beta}y}\right\},$$

$$\text{for} \quad |y|<y_0\,, \qquad \text{where} \quad \beta\equiv 1+\alpha^2\ \frac{g^2}{e^2}\,; \tag{34}$$

$$\begin{aligned}\tilde{\mathcal{A}}_\mu(p,y)&=g^2\,\tilde{J}_\mu(p)\,c\exp(-py), \quad \text{for} \quad y>y_0\\ \tilde{\mathcal{A}}_\mu(p,y)&=g^2\,\tilde{J}_\mu(p)\,c\exp(py), \quad \text{for} \quad y<-y_0,\end{aligned} \tag{35}$$

where b,d and c are momentum-dependent constants which can be determined by matching the solutions and their derivatives at $y=\pm y_0$.

In addition, we get that $\partial_y\tilde{\mathcal{A}}_y=0$. Our primary interest is the solution confined to the wall. This is determined by Eq. (34) near the point $y=0$,

$$\tilde{\mathcal{A}}_\mu(p,0)=g^2\ \tilde{J}_\mu(p)\left\{\frac{1}{2\sqrt{\beta}p}+\frac{1}{2\sqrt{\beta}p}\frac{(\sqrt{\beta}-1)\exp(-\sqrt{\beta}y_0p)}{\mathrm{ch}(\sqrt{\beta}y_0p)\ +\ \sqrt{\beta}\ \mathrm{sh}(\sqrt{\beta}y_0p)}\right\}. \tag{36}$$

Taking the limit (33) we find

$$\tilde{\mathcal{A}}_\mu(p,y=0)\ =\ \frac{\tilde{J}_\mu(p)}{p^2+pp_*}\,, \tag{37}$$

i.e. precisely the expression which we analyzed in the previous section. Thus, the vector fields are quasilocalized due to the induced kinetic term on the domain wall.

4 Some Phenomenological and Astrophysical Implications

Let us first discuss the mass scale relevant to five- and four-dimensional physics. The mass parameter relevant to five-dimensional physics is g^{-2}. Since the five-dimensional theory under consideration is an effective one, a five-dimensional cut-off Λ_5 must be introduced. The loop expansion in the five-dimensional theory runs in powers of $g^2\Lambda_5/4\pi$. For the theory to make sense we must require that $\Lambda_5 \sim g^{-2}$. On the other hand, the scale of g^{-2} is set from requiring deviations from the Coulomb law in our wall-confined world to be acceptable, i.e. parameter p_* be small enough. Barring the possibility of exceedingly large N_f one can say that p_* and g^{-2} are of the same order of magnitude.

Note that the four-dimensional world the expansion runs in $p_*/M = 1/(g^2M)$ where the parameter M is related to the masses of the localized modes. In this way this is the strong coupling expansion with respect to the five-dimensional theory. However, the above constraints are evaded if one adopts a scenario where the charged matter fields are confined to a brane and do not propagate in the bulk. Then, in the case of the Abelian gauge field, there are no loops in the bulk, all loops are confined to the wall world-volume[6].

Let us discuss some bounds on the crossover scale r_c. At distances $r > r_c$ the electrodynamics we consider becomes five-dimensional. At first sight, the bound on r_c is expected to be very severe, at least comparable to the present Hubble size. This is due to the fact that we are constantly detecting electromagnetic waves propagating over the cosmic distances, and these waves seem to behave in a perfectly four-dimensional way.

Surprisingly enough, the actual bound on r_c is rather mild, as we explain below. This is due to the phenomenon of an "infrared transparency" according to which the large wavelength radiation penetrates easier in extra dimensions. To illustrate the point let us consider an electromagnetic wave produced by a monochromatic source l located on the brane

$$J_\mu(x,y) \ \sim \ l_\mu \ \delta(y) \ \delta^{(3)}(x) \ \exp(i\omega t)\,. \tag{38}$$

The corresponding wave equation is given in (30) where the right-hand side is substituted by (38) and $\mathcal{A}_y$ is put to zero. Qualitatively, the wave behavior can be understood as follows. If it were not for the brane world-volume contribution (the second term on the left-hand side of Eq. (30)) the wave would behave as five-dimensional,

$$\sim \ \sqrt{\omega} \ \frac{e^{i\omega(t-R)}}{R^{\frac{3}{2}}}\,. \tag{39}$$

[6]Another option, which we will not pursue here, is to make g^2 defined by a scalar field ("dilaton") which has a y dependent profile so that g^2 is large on the brane while it is suppressed away from the brane.

where R stands for the five-dimensional radial coordinate. On the other hand, a four-dimensional wave which propagates in the world-volume would be described by

$$\sim \frac{e^{i\omega(t-R)}}{R} \, \delta(y) \; . \tag{40}$$

Using these expressions and Eq. (30) one can estimate a distance in the brane world-volume at which the crossover between the four-dimensional and five-dimensional behavior should take place,

$$r \; \sim \; r_\omega \; \sim \; \omega r_c^2 \; . \tag{41}$$

Thus r_ω sets the distance below which the wave propagates as four-dimensional. Surprisingly enough, this resembles the behavior of the gravitational waves in [15]. For short wave length physics becomes exceedingly more four-dimensional. The waves with the frequency $\omega \gg 1/r_c$ will propagate as four-dimensional waves over the distances much larger than r_c. This suggests that even if the Coulomb law gets modified at relatively short distances, propagation of visible light at larger scales will still look perfectly four-dimensional. For instance, let us assume that the Coulomb law breaks down beyond the solar system size, that is to say $r_c \sim 10^{15}$ cm. The largest wave-length radiation propagating over cosmic distances which has been detected so far are the radio waves in a meter wave-length range. Such radiation would propagate according to the laws of four-dimensional physics over the distance scale

$$r_{\rm radio} \sim 10^{28} \text{ cm} \; . \tag{42}$$

This is comparable to the size of the Universe. Thus, the Coulomb law might break down at a scale of the solar system size and we would not even notice it!

By the same reason, astrophysical processes, such as star cooling, cannot provide any significant bound on r_c. The photons emitted by stars will remain four-dimensional over the scales larger that the present Hubble distance. In this respect, the radio astronomy may provide a useful bound on r_c through precision measurements of the frequency-on-intensity dependence of the large wave-length radiation.

Another prediction of the present scenario, is a possibility of the charge non-conservation on the brane. Non-conservation of global charges is a generic feature of brane world scenarios [19]. However, local charges, the ones that are associated with the unbroken local symmetries are expected to be conserved in the world-volume due to the flux conservation arguments. This takes place in the scenario of Ref. [10] where photon is strictly localized on the brane due to the condensation of magnetic charges in the bulk. As a result, the electric flux is repelled from the exterior and spreads on the brane, producing a four-dimensional Coulomb potential between the two brane-localized sources. In such a scenario, if one tries to remove a Q-amount of the electric charge from the brane, the flux tube will be stretched between the brane and the particle. The throat of the flux tube on the brane will be seen by the

brane observer as a charged particle of the same charge Q and no apparent charge violation will be present. The flux tube may break into charge-anticharge pairs, and neutral states can escape into extra dimensions. However, the end of the flux tube that will remain attached to the brane will still carry the same charge Q.

In our present scenario photons may leak in extra dimensions, and thus, electric charge can leak there too. Both of these effects give new striking features, very different from the standard brane-world scenarios in which gauge fields are localized on the brane.

One may think of various astrophysical implications of the photon disappearance in extra dimensions. For instance, there is a certain observational evidence [20] that distant supernovae appear to be dimmer than it is expected in the standard matter-dominated cosmological scenario. The results of these studies show that the distant supernovae are fainter. In the conventional four-dimensional theory this would mean that they are more distant than expected for the decelerating Universe, and, thus, indicate that the Universe is accelerating. In our case, however, this may mean that photons spread to extra dimensions.

A natural question in our framework is whether or not the dimmer supernovae can be simply the consequence of the fact that photons emitted by these stars dissipate into the fifth dimension on their way to the Earth? Unfortunately, this idea is not very easy to realize. For instance, the immediate question would be: What prevents the cosmic microwave background radiation (CMBR) from dissipating into the extra dimension? (The CMBR wave-length is much longer than that for the radiation detected from distant supernovae[7].) In this respect it may be interesting to find a scenario in which the resonance photon has a decay width which vanishes below some frequency ω_0 and is nonzero for higher frequencies. Assuming that ω_0 is less than the visible light frequency (10^6 m^{-1}), but is above the frequency of CMBR (mm^{-1}), the CMBR photons would not be able to escape in the bulk. On the other hand, the photons emitted by the distant supernovae are more energetic and would dissipate in extra dimensions. Of course, even in this case it is not at all clear that the frequency dependence on the disappearance rate can correctly fit into the observed supernovae spectrum, so that a more careful study is required.

Another comment concerns the galactic magnetic fields. These are used to put constraints on photon mass. In our case, however, we deal with a mild power-law modification of the Coulomb law and the use of galactic magnetic fields to constraint r_c would only be possible if detailed properties and the origin of these fields were known. Unfortunately, this information is not available at present (see, e.g., [21]).

5 A Byproduct Application?

One may try to exploit the very same idea of (quasi)localization of the gauge fields within the framework of Kaplan's suggestion [22] to generate chiral fermions on the

[7]We thank V. Rubakov for commenting on this issue.

lattice by considering five-dimensional lattice with a domain wall to which fermion zero modes are confined. As long as these wall-confined fermions carry color quantum numbers, they will generate, through the loops a kinetic term to the gauge fields which will be peaked on the four-dimensional wall. If the lattice parameters are appropriately chosen, the above kinetic term may force localization of the gauge fields on the same wall.

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